hep-ph/9409222
August 1994

QCD corrections to inclusive $\Delta S = 1, 2$ transitions

Matthias JAMIN*
Division TH, CERN, 1211 Geneva 23, Switzerland

ABSTRACT

The talk summarises a calculation of the two-point functions for $\Delta S = 1$ current-current and QCD-penguin operators, as well as for the $\Delta S = 2$ operator, at the next-to-leading order.[1] The size of the gluonic corrections to current-current operators is large, providing a qualitative understanding of the observed enhancement in $\Delta I = 1/2$ transitions. In the $\Delta S = 2$ sector the QCD corrections are quite moderate ($\approx -20\%$). This work has been done in collaboration with Antonio Pich.

1. Introduction

Standard Model calculations of non-leptonic weak decays, conventionally being performed in the framework of the operator product expansion, are subject to strong interaction corrections, both for the short-distance contributions (coefficient functions) as well as for the long-distance part (matrix elements). Whereas the coefficient functions are evaluated at a high scale $\mathcal{O}(M_W)$, and therefore calculable in perturbation theory, the matrix elements receive contributions from low scales $\mathcal{O}(\Lambda_{QCD})$ and hence have to be calculated in some "non-perturbative" framework.

The situation very much changes if one considers weak decays at the inclusive level. At the inclusive level, the quantity of interest is the two-point function of two effective Hamiltonians, which can be investigated with perturbative QCD methods. In the following, we shall restrict ourselves to the $\Delta S = 1$ Hamiltonian[2]

$$\mathcal{H}_{\rm eff}^{\Delta S=1} \;=\; \frac{G_F}{\sqrt{2}}\, V_{ud}V_{us}^{*} \sum_i C_i(\mu^2)\, Q_i \,, \tag{1}$$

obtained through the operator product expansion, and consider the two-point function

$$\Psi^{\Delta S=1}(q^2) \;\equiv\; i\int dx\, e^{iqx}\, \langle 0|\, T\{\, \mathcal{H}_{\rm eff}^{\Delta S=1}(x)\, \mathcal{H}_{\rm eff}^{\Delta S=1}(0)^{\dagger}\}|0\rangle \,. \tag{2}$$

The associated spectral function $\Phi(s) \equiv \frac{1}{\pi}{\rm Im}\Psi^{\Delta S=1}(s)$ is a quantity with definite physical information. It describes in an inclusive way how the weak Hamiltonian couples the vacuum to physical states of a given invariant mass. General properties like the observed enhancement of $\Delta I = 1/2$ transitions can be then rigourously analysed at the inclusive level.[3]

*Invited talk presented at the conference "QCD'94", Montpellier, France, July 7 - 13, 1994. To appear in the proceedings.

2. Current-current operators

The two-point functions for the current-current operators are most conveniently calculated in the diagonal basis in which the two operators Q_+ and Q_- do not mix. The scheme- and scale-independent spectral functions are then found to be[1]

$$\Phi_{++}(s) = \theta(s)\,\frac{8}{15}\,\frac{s^4}{(4\pi)^6}\,\alpha_s(s)^{-4/9}\,C_+^2(M_W^2)\Big[1-\frac{3649}{1620}\,\frac{\alpha_s(s)}{\pi}\Big], \tag{3}$$

$$\Phi_{--}(s) = \theta(s)\,\frac{4}{15}\,\frac{s^4}{(4\pi)^6}\,\alpha_s(s)^{8/9}\;\;C_-^2(M_W^2)\Big[1+\frac{9139}{810}\,\frac{\alpha_s(s)}{\pi}\Big], \tag{4}$$

where $C_\pm(M_W^2)$ are the coefficient functions corresponding to the operators $Q_\pm$.[4] All actual calculations have been performed in two different schemes for γ_5, an "naively" commuting γ_5 and the algebraically consistent non-anticommuting γ_5 according to 't Hooft and Veltman,[5] and we have explicitly verified the scale- and scheme-independence of physical quantities.

Taking $\alpha_s(s)/\pi \approx 0.1$, at the NLO we find a moderate suppression of Φ_{++} by roughly 20%, whereas Φ_{--} acquires a huge enhancement of the order of 100%. Because Φ_{++} solely receives contributions from $\Delta I = 3/2$, and Φ_{--} is a mixture of both $\Delta I = 1/2$ and $\Delta I = 3/2$, this pattern of the radiative corrections entails a strong enhancement of the $\Delta I = 1/2$ amplitude. Hence, we are provided with a very promising picture for the emergence of the $\Delta I = 1/2$–rule.

3. Full result including penguins

In the process of mixing of operators under QCD corrections, four additional operators, the so called "penguin-operators" are generated. At the leading order, the resulting effective Hamiltonian can still be diagonalized, but at the next-to-leading order, this is no longer possible. So, in this case, the two-point function of the set of operators is a 6×6-matrix.

Following the notation of ref. [2], the coefficient functions for $\Delta S = 1$ weak processes can be decomposed as $C(s) = z(s) + \tau\, y(s)$, where $\tau \equiv -\,(V_{td}V_{ts}^*)\,/\,(V_{ud}V_{us}^*)$. The coefficient function $z(s)$ governs the real part of the effective Hamiltonian, and $y(s)$ parametrises the imaginary part and governs, e.g., the measure for direct CP-violation in the K-system, ε'/ε. We thus can form two spectral-functions $\Phi_{z,\,y}(s)$ corresponding to z and y respectively.

In the region $s = 1-10\,\mathrm{GeV}^2$, relevant for example in QCD sum rule calculations of the K-system, and for a central value $\Lambda_{\overline{MS}} = 300\,\mathrm{MeV}$, the radiative QCD correction to Φ_z ranges approximately between 40% and 120%, whereas in the case of Φ_y we find a correction of the order of 100%–240%.[1] Because the two-point function is constructed as the square of the effective Hamiltonian, the actual corrections to $\mathcal{H}_{\mathrm{eff}}^{\Delta S=1}$ are only about half the corrections to the 2–point function. Therefore, the perturbative QCD correction to the real part of the effective Hamiltonian turns out to be 20%–60%, and for the imaginary part 50%–120%.

4. The $\Delta S = 2$ operator

For the case of $\Delta S = 2$ transitions, things are somewhat simpler because there is only one operator. In 4 dimensions this operator is given by

$$Q_{\Delta S=2} \;\equiv\; (\bar{s}d)_{\mathrm{V-A}}\,(\bar{s}d)_{\mathrm{V-A}}\;. \tag{5}$$

Calculating the spectral function $\Phi_{\Delta S=2}(s)$ for $Q_{\Delta S=2}$, we obtain

$$\Phi_{\Delta S=2}(s) \;=\; \theta(s)\,\frac{32}{15}\,\frac{s^4}{(4\pi)^6}\,\alpha_s(s)^{-4/9}\Big[\,1-\frac{3649}{1620}\,\frac{\alpha_s(s)}{\pi}\,\Big]\,. \tag{6}$$

Because both Q_+ and $Q_{\Delta S=2}$ have the same chiral representation, as expected, apart from a global factor, their spectral functions agree. We observe that the NLO QCD-correction is negative and of the order of 20%, for $\alpha_s(s)/\pi \approx 0.1$.

5. Conclusion

Our work improves and completes the two-point function evaluation of ref. [3] with two major additions: the recently calculated NLO corrections to the Wilson-coefficient functions have been taken into account and, in addition, we have incorporated previously missing contributions from evanescent operators. The final results are then renormalization scheme- and scale-independent at the NLO, and, therefore, constitute the first complete calculation of weak non-leptonic observables at the NLO, without any hadronic ambiguity.

The structure of the radiative corrections to two-point functions of $\Delta S = 1$ and $\Delta S = 2$ operators also allows for a deeper understanding, why a description of non-leptonic weak decays in terms of diquarks was so successful as far as the $\Delta I = 1/2$–rule is concerned.[6] The explicit calculation shows that quark-quark correlations give the dominant contribution to the QCD corrections, and through working with effective diquarks, these contributions can be summed up to all orders.

A full QCD calculation has been possible because of the inclusive character of the defined two-point functions. Although only qualitative conclusions can be directly extracted from these results, they are certainly important since they rigourously point to the QCD origin of the infamous $\Delta I = 1/2$–rule, and, moreover, provide valuable information on the relative importance of the different operators, which can be very helpful to attempt more pragmatic calculations.

Acknowledgements The author would like to thank S. Narison for the invitation to the conference and A. Pich for most enjoyable collaboration.

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